

Modelling an Imperfect Market

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Abstract

We propose a simple market model where agents trade different types of products with each other by using money, relying only on local information. Value fluctuations of single products, combined with the condition of maximum profit in transactions, readily lead to persistent fluctuations in the wealth of individual agents.

I. INTRODUCTION

Financial markets usually consist in trades of commodities and currencies. However, one can easily find cases in other types of human endeavour that parallel the activities observed in financial markets. For example, a politician may select the standpoints in his/her platform

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in exchange for votes in the coming election [1]. Also, scientists may trade ideas in order to generate citations. However, the easiest quantifiable marketplace is the financial one, where the existence of money allows a direct measure of value.

There have been several proposals to model such markets. A recent review was presented by Farmer [2]. Bak, Paczuski and Shubik [3] have proposed a model where they consider the price fluctuations of a single product traded by many agents, all of which use the same strategy. In that model fat tails and anomalous Hurst exponents appear when global correlation between agents are introduced. An alternative model, presented by Lux and Marchesi [4], is driven by exogenous fluctuations in “fundamental” values of a single good, with induced non-gaussian fluctuations in price assessment of this good arising from switching of strategies (trend followers and fundamentalists) by the agents.

Below we give a few considerations that had led us in the development of a model where the dynamics arises solely from the interaction of agents trying to exchange several types of goods.

Financial markets exhibit a dynamical behavior that, even in the absence of production, allow people to become either wealthy or poor. If in these markets there were some sort of equilibrium, e.g. due to complete rationality of the players, this would not be possible. In order to create wealth or bankruptcy, people have to outsmart each other. This means that each trader attempts to buy as cheaply and sell as expensively as possible. This demands that an agent should find a seller which sells at a low price, and later another one that is willing to buy the same product at a higher one. Thus different agents price differently the same product. In this work we demonstrate that it is possible to devise a simple model for such a non-equilibrium market. We call it the Fat Cat (FC) model, as it functions on greed (each agent buys to optimize his own assets) and creates a market with large fluctuations, i.e. fat tails. In Sect. II we present this model and show examples of its dynamical evolution. We show that it leads to an ever fluctuating market. In the following Sect. III, the time series generated by this model is analyzed and it is shown that it exhibits persistency in the fluctuations of the wealth of individual agents. In the final section we summarize the

work and present suggestions for generalizing the model to let individual agents evolve their individual trading strategies.

II. DESCRIPTION OF THE FC MODEL

Consider a market with N_{ag} agents, each having initially a stock of N_{un} units of products selected among N_{pr} different types of products. In a previous work [5] we have shown that, for the case of agents having memory of past transactions, such a market spontaneously selects one of the products as the most adequate as a means of exchange. The product so chosen acts as money in the sense that it is accepted even when the agent does not need it, because through the memory of past requests of products the agent knows it is in high demand, and therefore it will be useful to have it to trade for other products. The selection of a product as an accepted means of exchange is not indefinite: after some time another one substitutes it as the favorite currency. The time scale for these currency substitutions is large when measured in number of exchanges between individual agents.

In the present model we consider that each of the agents initially has N_{mon} units of money. According to the discussion in the preceding paragraph, in principle money could be viewed as one of the products, but here we consider it a separate entity in order to quantify prices. The N_{un} products given initially to each agent are selected randomly. Later, during the time evolution of the system, each agent i has, at each time step, an amount of money $M(i)$, $i = 1, \dots, N_{ag}$, and a stock of the different products j , $S(i, j)$, where $j = 1, \dots, N_{pr}$. Since the model uses money as means of exchange, agents assign different prices to the different products in their possessions. The prices of the different items in the stock of agent i , $P(i, j)$, are taken initially to be integers uniformly distributed in the interval $[1, 5]$. We have verified that the evolution of the system does not depend on this particular choice.

How do we picture such a market? We may imagine antique collectors trying to buy objects directly from each other, using their own estimates for the prices of the different stock. When two agents meet, one of them, the buyer, checks the seller's price list, and

compares it with his own price list. We have chosen the antique collector market as an example because few other markets show spatial price fluctuations at such a high level. The decision to buy or not, and the changes in the value of the agents' products are given by some strategy, which for now assume is the same for all agents in the system. Among all the products that the buyer considers to be possible buys (having a price set by the seller which is lower than the one he would sell the same product for), he will single out the best one, and will then attempt to buy it. But, if the buyer finds no products that he considers as good buys, the seller will consider that he has overpriced his goods and will as a consequence tend to lower his prices. At the same time, the buyer will think that his price estimate was too low, and as a consequence raise his price estimate.

This is the basis for our computer simulation for such a market place. In it, we assume that at each time step the following procedure takes place:

1. Buyer (b) and seller (s) are selected at random among the N_{ag} agents. If the seller has no products to offer, then another seller is chosen.
2. The buyer selects the product j in the seller's stock which maximizes $P(b, j) - P(s, j)$ (i.e. his profit). The corresponding j , we call j_{bb} (best buy).
3. If the buyer does not have enough money, (i.e. if $M(b) < P(b, j_{bb})$), we go back to the first step, choosing a new pair of agents.
4. If the buyer has enough money we proceed.

If $P(s, j_{bb}) < P(b, j_{bb})$, the transaction is performed at the seller's price. This means that we adjust: $S(b, j_{bb}) \rightarrow S(b, j_{bb}) + 1$, $S(s, j_{bb}) \rightarrow S(s, j_{bb}) - 1$,

$M(b) \rightarrow M(b) - P(s, j_{bb})$, $M(s) \rightarrow M(s) + P(s, j_{bb})$.

5. If $P(s, j_{bb}) \geq P(b, j_{bb})$, the transaction is not performed.

In this case, the seller lowers his price by one unit,

$$P(s, j_{bb}) \rightarrow \max(P(s, j_{bb}) - 1, 0),$$

and the buyer raises his price by one unit,

$$P(b, j_{bb}) \rightarrow P(b, j_{bb}) + 1.$$

We see that, according to these rules, buyer and seller decide on a transaction based only on their local information, i.e. their estimates of the prices for the different products they possess. These prices are always non-negative integers. Also note that since, as defined in step 3 above, the price offered by the buyer cannot be higher than the amount of money he has, we are not allowing for the agents to get in debt. Further, the model tends to equilibrate large price differences, according to step 5, but induces price differences when buyer and seller agree on the price of the most tradeable product. This non-equilibrating step is essential to induce dynamics in a model like the present one, where all agents follow precisely the same strategy. Without it the system would freeze into a state where all agents agree on all prices.

The rules given above are just one possible set of rules for transactions. We have found other sets that lead to a behavior qualitatively similar to the one shown below for the present rules.¹ We now show that, under this set of local rules, the distribution of wealth organizes itself into a dynamically stable pattern, and the same phenomenon takes place with the prices.

One should emphasize that there is not an accepted market value for the products. Indeed, due to the price adjustments performed in unsuccessful encounters, the prices never reach equilibrium, and different agents may assign different prices for the same product.

¹A quote from Marx is appropriate here: “These are my principles. If you do not like them, I have others” [6].

In Fig. 1 we illustrate this point, showing the price assigned by two different agents to the same product, as a function of time. Time is defined here in terms of the number of encounters between agents, and one time unit is the average time between events where a given agent acts as a buyer. We note that during a considerable fraction of the time there is a relatively large difference between the prices assigned by the agents. This shows that there is a margin for making profit in such a market, i.e. arbitrage is possible. We have verified that the average market price of a good fluctuates with a Hurst exponent of ~ 0.5 .

In Fig. 2 we show, for the same time interval, an example of the evolution of key quantities in the model associated to Agent 1 in Fig. 1. The total wealth of an agent i is the amount of money plus the value of all goods in the agent's possession:

$$w(i) = M(i) + G(i) . \quad (1)$$

Here the value of product j is defined as the average of what all agents consider its value to be:

$$P_{ave}(j) = \frac{1}{N_{ag}} \sum_{i=1}^{N_{ag}} P(i, j) , \quad (2)$$

and the value of all agent i 's goods $G(i)$ is then defined as

$$G(i) = \sum_j S(i, j) P_{ave}(j) . \quad (3)$$

We note that there are considerable fluctuations in the wealth of this agent. The study of these fluctuations is essential to the understanding of the properties of the model, and we develop this in the following section.

III. FLUCTUATIONS IN THE FC MODEL

In order to quantify the fluctuations in wealth, we show in Fig. 3 the RMS fluctuations of the wealth of a selected agent as function of time. The figure illustrates that the wealth fluctuations can be characterized by a Hurst exponent [7] $H \approx 0.7$. We have examined variants of both model parameters and rules to check the stability of this result. We found it

to be stable, as long as one keeps greed in the model. For example, if one reduces the number of product types to only $N_{pr} = 2$ (keeping $N_{ag} = 100$, $N_{mon} = 500$ and $N_{un} = 100$ initially) the Hurst exponent remains unchanged although the scaling regime shrinks. Similarly setting $N_{un} = 2$ (with $N_{pr} = 100$, $N_{ag} = 100$ and $N_{mon} = 500$) resembling antique dealing where each agent owns a few of many possible products, also lets the Hurst exponent unchanged. On the other hand, if greed is removed from the model, e.g. the buyer selects a product at random from the seller's store, without consideration to the profit margin, the Hurst exponent drops to 0.5, signaling that no correlations develop in such a case.

Thus, the optimization of product selection expressed by step 2 in the procedure is closely related to the persistent fluctuations seen in our model. We have checked that other optimization procedures, as e.g. selecting the cheapest product or the product the buyer has the least of in stock also give similar persistent fluctuations. Oppositely, random selection reflects an economy where different products do not interact significantly with each other, and where our market of N_{pr} different types of products nearly decouples into N_{pr} different markets. With random selection, the only interaction between products is indirect; it appears due to the constraint of the agents' money taking only non-negative values. Overall, the random strategy gives a less fluctuating market where agents agree more on prices. Greed indeed makes our model world both richer and more interesting (which is *not* to say *better*).

We now try to quantify these wealth fluctuations. Fig. 4 displays the changes in the value of w for several time intervals Δt . The three curves are histograms of wealth changes for respectively $\Delta t = 10$, $\Delta t = 100$ and $\Delta t = 1000$. One observes fairly broad distributions with a tendency to asymmetry in having bigger probability for large losses than for large gains. Similar skewness is seen in real stock market data. Furthermore, the tails are outside the Gaussian regime [10]. In the upper panel of Fig. 5, this is investigated further by plotting the histograms as function of the logarithmic changes in w , $r_{\Delta t} = \log_2 w(t + \Delta t) - \log_2 w(t)$ (*log-returns*), for the same three time intervals. In that figure we have collapsed the curves onto each other by rescaling them using the Hurst exponent $H = 0.69$, consistent with the one found in Fig. 3. For the two short time intervals the collapse is nearly perfect, even in

the non-Gaussian fat tails. For larger time intervals the distribution changes from a steep power law or stretched exponential, to an exponential, and finally becomes Gaussian for very large intervals (not shown as it is very narrow on the scale of this figure). We think it is interesting to notice that our model is consistent with the empirical observation that in real markets the probability for large negative fluctuations is larger than that for large positive ones.

In the lower panel of Fig. 5 we examine in details the fluctuations for $\Delta t = 1$, and compare the fat tails with truncated power law decays $P(r) \sim 1/r^4 \cdot \exp(-|r|/R)$ which for such small time intervals is nearly symmetrical. The $1/r^4$ is consistent with the fat tail observed on 5-minute interval trading of stocks [10]. The cut off size $R = 0.8$ corresponds to cut offs when price changes are about a factor 2 from the original price, a regime which is not addresses in the short time trading analysis of [10]. We stress that our analysis of fat tails includes a wide distribution of wealth, thus large relative changes of wealth are presumably mostly associated to poor agents. Thus the seemingly good fit to fat tails observed for stock market fluctuations in the 1000 largest US companies [10] may be coincidental.

IV. SUMMARY AND DISCUSSION

The appearance of fat tails [8–10] and Hurst exponents [9,11] larger than 0.5 in the distribution of monetary value appears to be a characteristic of real markets. The present model is, as it was the case with the previous version [5], qualitatively consistent with these features. We stress that here, as for the previous model, we are not including any development of strategy by the agents that might force the emergence of cooperativity [12,13]. A more important difference to game theoretic models is that the minority game, as well as the evolving Boolean network of Paczuski, Bassler and Corral [13] evolves on basis on a global reward function. With the present model we would like to open for models which evolve with imperfect information, preferably in a form which allows direct comparison with financial data. The present model does this, and in a setting where there are many products

and thus possibilities for making arbitrage along different “coordinates”.

Compared to models with fat tails or persistency arising from boom-burst cycles, as the trend enhancing model of Delong [15] or the trend following model of Lux and Marchesi [4], the present model discuss anomalous scaling in a market where no agent has a precise knowledge of the global or average value of a product. There is only local optimization of utility (estimated market value) and all trades are done locally without the effects of a global information pool. This imperfect information gives a possibility for arbitrage and opens for a dynamic and evolving market.

The model we propose is for a market composed of agents, goods and money. We have demonstrated that such a market easily shows persistent fluctuations of wealth, and seen that this persistency is closely related to having a market with several products which influence each others trading. As seen in Fig. 2, wealth increase of an agent is associated with active trading of few products. This may be understood as follows: when the number of options a seller presents is small, it is hard for the buyer to find a bargain, so if a transaction is performed it will probably be at a good price for the seller.

The simpler model of Ref. [5], had persistency in the fluctuations in the demand for different products, whereas, as mentioned above, the persistence here is in the fluctuations in the wealth of the different agents. It is interesting to mention that the Hurst exponents in the two models take very similar values, and this for a wide variety of the respective parameters. There is a kind of duality in that in the simpler model a product increased in value when it was held by relatively few agents, whereas in the present model the agents increase their wealth by specializing to few products.

The setup proposed here with agents and products with individual local prices allows for also different individual strategies of the agents. For example both the selection of greed versus random strategy under step 2, and the particular adjustment of values defined under step 4-5 could be defined differently from agent to agent. One may accordingly have different strategies for different agents, and each agent could change its strategy in order to improve his performance. The evolution of these strategies would then become an inherent part of

the dynamics. This opens for evolution of strategies as part of the financial market, and will be discussed in a separate publication [14].

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FIGURES

FIG. 1. Selling prices of a specific product by two different agents, as a function of time. The simulation was performed for the following parameter values: $N_{ag} = 100$, $N_{pr} = 100$, $N_{mon} = 500$ and $N_{un} = 100$, which apply to the calculations presented in all figures in this work.

FIG. 2. Amount of money M , number of different products and combined wealth $w = M + G$, held by an agent as a function of time. See text for further details.

FIG. 3. RMS fluctuations of the wealth of a given agent as function of time interval Δt . In order to guide the eye we also plot the power function $\Delta t^{0.7}$.

FIG. 4. Probability of having changes in wealth $\Delta w = w(t + \Delta t) - w(t)$ as a function of their size, for the three different time steps $\Delta t = 10$ (full line), $\Delta t = 100$ (long dashed line), and $\Delta t = 1000$ (short dashed line).

FIG. 5. a) Similar to Fig. 4 except that wealth fluctuation is measured here in units of $\log_2 [w(t + \Delta t)/w(t)] / \Delta t^H$, where Δt takes the same values as above, and we took for the Hurst exponent the value $H = 0.69$. b) Fits to the probability of having a wealth loss (gain) as a function of the log-returns $r = \log_2 [w(t + \Delta t)/w(t)]$, for the case $\Delta t = 1$. The fit $P(r) = 5/(r^2 + 0.03)^2 \cdot \exp(-|r|/0.4)$ have asymptotic expressions for large $|r|$ of the form $P(r) \sim 1/r^4 \cdot \exp(-|r|/R)$, with $R = 0.4$.









